Approved For Release 2009/08/17 : CIA-RDP88-00904R000100100

Third United Nations International Conference on the Peaceful Uses of Atomic Energy

A/CONF.28/P/329
USSR
May 1964
Original: RUSSIAN

Confidential until official release during Conference

THEORETICAL MODEL FOR TURBULENT TRANSFER IN THREE-DIMENSIONAL FLUID FLOW

N.I.Buleev

The aim of this paper is to develop methods of calculating the velocity and temperature fields for turbulent flows in the channels of arbitrary shape.

§I. Equations of turbulent motion for averaged values

Let us write the system of equations of turbulent motion of an incompressible fluid for averaged values in Cartesian coordinates.

Equations of motion

$$\frac{\partial \rho \bar{v}_k}{\partial t}+\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\rho \bar{v}_i \cdot \bar{v}_k)=-\frac{\partial \bar{p}}{\partial x_k}+\rho\nu\Delta\bar{v}_k-\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\rho\overline{v_i' v_k'}) \qquad (I.I)$$

the equation of conservation of mass

$$\sum_{k=1}^{3}\frac{\partial}{\partial x_k}(\rho\bar{v}_k)=0, \qquad (I.2)$$

the equation of heat inflow

$$\frac{\partial \rho\bar{T}}{\partial t}+\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\rho\bar{v}_i\bar{T})=\rho\kappa\Delta\bar{T}-\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\rho\overline{v_i' T'}). \qquad (I.3)$$

Here v_1, v_2, v_3 - are velocity components, p is the pressure, T is the temperature, ρ is the fluid density, ν is kinematic viscosity and k is the coefficient of temperature conductivity. The symbol Δf means the three-dimensional Laplace operator. For the sake of simplicity of writing down the equations (I.3)-(I.4) the coefficient of viscosity $\mu=\rho\nu$ and the coefficient of heat conductivity $\lambda=c\rho\kappa$, are assumed to be constant.

After the appearance of the Reynolds work [I] in which the equations (I.I)-(I.2) have been obtained, the further development of turbulent transfer theory procceded by way of building of semiempirical models, the basis of which was the concept of a"mixing" length" similar to one

of the mean free path of the molecules in the kinetic theory of gases (see, for example, the survey of the monograph [2]).

In the analysis of one-dimensional fluid flow $w(x)$ the Prandtl-Karman relation is widely used in practice as a semiempirical approximation for the turbulent stresses $\rho \overline{u'w'}$ which has a form

$$-\rho \overline{u'w'} = \rho \varepsilon \frac{\partial w}{\partial x} \qquad \text{where} \quad \varepsilon = \ell^2 \left| \frac{dw}{dx} \right|, \qquad (I.4)$$

ℓ being a turbulent scale ("mixing length").

If one takes $\ell = \varkappa x$ and an appropriate origin for x, Prandtl relation for the coefficient ε ensured in many cases a quite satisfactory solution of the hydrodynamical problems.

The statistical theory of turbulence was developed later (in twenties-thirties). In spite of the serious theoretical successes the statistical theory could not give a practically acceptable procedure for completing the Reynolds equations.

In this paper an extension of the semiempirical Prandtl-Karman theory is made to the case of an arbitrary three-dimenthional fluid flow.

§2. Three-dimensional model of turbulent transfer in arbitrary fluid flow

We shall consider the turbulent motion as a superposition of irregular transient vorticities on the main averaged motion. Every eddy which appears by chance and then quickly disappears and which has a diameter equal, say, to 2ℓ, transfers fluid lumps of ℓ size over the distances also approximately equal to ℓ.

For the sake of convenience of mathematical description of the momentum and heat transfer caused by random motions of fluid lumps inside of transient vorticities we shall suppose that fluid lumps fly out from the vicinity of every flow point M condidered in the coordinate system that moves with the averaged flow velocity. This flying-out takes place in all directions with equal pro-

bability. Introduce the notion of a linear scale of turbulence $L(M)$ which characterizes the linear size of transient vorticities in the vicinity of a current point M of fluid flow. We shall suppose that the characteristic "diameter" d of lumps which fly out from the vicinity of point M is identical to the scale L with an accuracy of a constant factor β :

$$d = \beta L. \tag{2.I}$$

Following the idea of the local similarity we shall suppose that the modulus of turbulent velocity of the lump occuring in the vicinity of M is proportional to the modulus of deformation of the velocity field of the averaged motion in M and the characteristic scale L in this point:

$$v'' = \begin{cases} \mu L \left| \frac{\partial v}{\partial r} \right|, & \text{if } \frac{L^2}{\nu} \left| \frac{\partial v}{\partial r} \right| \geq w, \quad (2.2) \\ 0, & \text{if } \frac{L^2}{\nu} \left| \frac{\partial v}{\partial r} \right| < w, \quad (2.3) \end{cases}$$

where

$$\left| \frac{\partial v}{\partial r} \right|^2 = 2\left(\frac{\partial \bar{u}}{\partial x}\right)^2 + 2\left(\frac{\partial \bar{v}}{\partial y}\right)^2 + 2\left(\frac{\partial \bar{w}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y}\right)^2 + \left(\frac{\partial \bar{w}}{\partial x} + \frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right)^2,$$

μ and w are dimensionaless constants.*)

Approximate formula for characteristic linear scale L will be done later.

Now we shall analyse the turbulent stresses $\tau_{ik} = -\rho \overline{v_i' v_k'}$ and the turbulent heat flows $q = c\rho \overline{v' T'}$ in equations(I.I-I.3). In every point M the quantities $\rho \overline{v_i' v_k'}$ and $c\rho \overline{v_i' T'}$ are respectively averaged fluxes of the momentum mv and of the heat cmT generated by the turbulent velocity component v_i' in the positive direction of the axis x through the unit

*) "Energy equation" (2.2) is similar in its structure to the expression for the kinetic energy dissipation per unit mass in a laminar fluid flow.

329

surface perpendicular to x_i and moving with the averaged flow velocity at M.

Consider a unit area at some point M inside the flow perpendicular to the x_i-direction. This surface is considered to move with a velocity of the averaged motion at M (Fig.I). Lumps from the vicinities of various flow points can pass through this surface. At some moment t let lump occuring in the vicinity of M pass the point M_0 with the velocity V. If the lump moves from M to M_0 without deceleration, its velocity V would be exactly equal to an algebraic sum of the averaged motion velocity $\vec{V}$ at M and the turbulence lump velocity V' taken in the coordinate system moving with the averaged flow at M. The turbulence velocity $v_i'(M_0)$ at the moment of passing of the lump would be accordingly equal to

$$v_i'(M_0) = V'(M)\cos(s,'x_i) + [\bar{v}_i(M) - v_i(M_0)], \tag{2.4}$$

where s' is the direction of velocity vector V' in the coordinate system moving with the averaged flow at M (Fig.I).

But in reality because of the interaction of the lump with the surroundings lump velocity in the process of its motion and therefore real turbulence velocity $v_i'(M_0)$ will be differed from that calculated from (2.4). Analogically at the moment of passing of the lump through M_0 the temperature of the lump considered will not be equal to its temperature at the origin M.

Introduce a hypothesis about the interaction of a moving lump with the surrounding fluid. The equation for the momentum and heat flow variation in a moving volume are written in the following manner:

$$dv_i^* = \frac{3}{R} A_1 (\bar{v}_i - v_i^*) dt, \quad dT^* = \frac{3}{R} A_2 (\bar{T} - T^*) dt \qquad {}^{*)} \tag{2.5}$$

Here v_i^* and T^* are velocity components and temperature in the moving volume $\bar{v}_i$ and $\bar{T}$ are those in the surrounding fluid, R is a "radius" of the lump, associated with the scale $L(M)$ by the relation (2.I), A_1 and A_2 are some coefficients of m/sec dimensions.

329

Make in (2.5) the transition from the independent variable t to the variable τ representing a distance between the moving lump and the origin , and it is in the system moving with the mean fluid flow velocity at M :

$$dt = \frac{d\tau}{v},$$

Then equations (2.5) take the form

$$\frac{dv_i^*}{d\tau} + P_1 v_i^* = P_1 \bar{v}_i \qquad \frac{dt^*}{d\tau} + P_2 T^* = P_2 \bar{T}, \tag{2.6}$$

where

$$P_1 = \frac{3A_1}{Rv'}, \qquad P_2 = \frac{3A_2}{Rv'} \tag{2.7}$$

Supposing the coefficients in (2.6) to be constant and the lump trajectory from M to M_0 to be rectilinear one can find approximate solution of equations (2.6).

In particular, the solution for $v_i^*(\tau)$ at $\tau = s$ where $s = MM_0$ is written as follows

$$v_i^*(s) = e^{-P_1 s}\left[\int_0^s P_1 \bar{v}_i e^{P_1 \tau'} d\tau' + v_i^*(0)\right], \tag{2.8}$$

where

$$v_i^*(0) = v'(M)\cos(s', x_i) + \bar{v}_i(M).$$

Write the velocity $\bar{v}_i$ in the surrounding fluid on the way MM_0 as a linear function of the distance τ . Then from (2.8) one can find

$$v_i'(M_0) = v_i^*(s) - \bar{v}_i(s) = v'(M) f_0 \cos(s', x_i) + [\bar{v}_i(M) - \bar{v}_i(M_0)] f \tag{2.9}$$

where $f_0 = e^{-P_1 s}$, $\qquad f_1 = \frac{1}{P_1 s}(1 - e^{-P_1 s})$.

Analogically we obtain an expression for the temperature fluctuation $T'(M_0)$ at the moment of passing of the lump considered through the point M.

$$T'(M_0) = [\bar{T}(M) - T(M_0)] f_2, \tag{2.10}$$

where $f_2 = \frac{1}{P_2 s}(1 - e^{-P_2 s}) = f_1(P_2 s)$.

We shall suppose that the difference of the averaged flow

---

* The introduction of equation (2.5) is an extension of ideas about an interaction between the moving lump and the surrounding fluid, developed in [7] , [8] , etc.

322

velocities $\vec{V}(M)-\vec{V}(M_0)$ at the distances MM_0 of the order of the mean free path of the lump is much less than the absolute value of the turbulence velocity V' of lump.

When we use the expressions (2.9–2.IO) we write the expressions for the flows $\rho V'(M_0)\, v_n'(M_0)$,

$\rho v_i'(M_0) \times \bar{\Psi}'(M_0)$.

which are formed at the moment of passing of the lump from the vicinity of M through M_0 as follows

$$\rho v_i'(M_0) v_k'(M_0) = \mathcal{F}_{ik}(MM_0) \simeq \rho V'^2(M) f_0^2 \cos(s, x_i)\cos(s, x_k) + \\ + \rho V'(M) f_0 f_1 [\bar{v}_k(M) - \bar{v}_k(M_0)] \cos(s, x_i) + \\ + \rho V'(M) f_0 f_1 [\bar{v}_i(M) - \bar{v}_i(M_0)] \cos(s, x_k), \qquad (2.II)$$

$$\rho v_i'(M_0) T'(M_0) = E_i(M, M_0) \simeq \\ \simeq \rho V'(M) f_0 f_2 [\bar{T}(M) - \bar{T}(M_0)] \cos(s, x_i). \qquad (2.I2)$$

In the expressions (2.I2–2.I2) the terms $[\bar{v}_i(M)-\bar{v}_i(M_0)][\bar{v}_k(M)-\bar{v}_k(M_0)] f_1^2$ and $[\bar{v}_i(M)-\bar{v}_i(M_0)][T(M)-T(M_0)] f_1 f_2$ are omitted as "small values of the higher order" and $\cos(s', x_j)$ is assumed to be equal to $\cos(s, x_j)$.

To obtain the averaged values $\rho \overline{v_i' v_k'}$ and $\rho \overline{v_i' T'}$ at M_0 it is necessary to perform the integration of the right-hand parts of (2.II–2.I2) over the surronding volume $\mathcal{D}$ with the appropriate weight function $f(M \to M_0)$ which is a probability of passing through the M_0 of the lump whose centre is out of the unit vicinity of an arbitrary point M.

$$\rho \overline{v_i' v_k'}(M_0) = \int_{\mathcal{D}} \mathcal{F}_{ik}(M, M_0) f(M \to M_0)\, d\tau, \qquad (2.I3)$$

$$\rho \overline{v_i' T'}(M_0) = \int_{\mathcal{D}} E_i(M, M_0) f(M \to M_0)\, d\tau. \qquad (2.I4)$$

We shall assume that the motion direction spectrum of the lumps passing throug the M_0 is approximately isotropic in the coordinate system moving with the velocity of averaged fluid flow at M_0.

We shall assume that the probability density $f(M \to M_0)$ has a form of spherically symmetrical normal distribution

329

law with the dispersion σ proportional to the turbulence scale L at M_o

$$f(M \to M_o) = \frac{1}{4\pi s^2} \sqrt{\frac{2}{\pi}} \, \frac{1}{\alpha L_o} e^{-\frac{1}{2} \cdot \frac{s^2}{(\alpha L_o)^2}}, \qquad (2.15)$$

where L_o is the value of the scale L at M_o and α is a dimensionless constant.

In the first approximation we shall suppose that the turbulent scale $L(M)$ in (2.1) and (2.15) is identical to the characteristic distance between M and the channel walls and determined by

$$\frac{1}{L} = \frac{1}{\pi} \int_{\Omega} \frac{e}{e} \, d\omega \qquad (2.16)$$

where ℓ is the distance between M and the channel wall in the direction w.

In the second approximation the scale $L(M)$ is assumed in addition to be dependent on local peculiarities of the velocity field of the averaged flow [4].

In the case of the forced fluid flows in closed channels it is assumed that the approximation (2.16) for the scale L is sufficient. The form of the function calculated from (2.16) for the channels of various sections is shown in §4. At this place we note that factor scale values L obtained from (2.16) are apart from a constant in good agreement with the experimental estimates of turbulence scales ([5], [6] etc.)

Thus, all the functions in the integral expressions are obtained. The expressions (2.23) and (2.14) are general integral formulas for the turbulent stresses tensor components and the turbulent flow vector components.

To simplify the use of the obtained approximations for $\rho v_i' v_k$ and $\rho v_i' T'$ make some further transformations for the expressions (2.13), (2.14).

The difference of U_i and of T at the points M and M_o is represented as an expansion along the coordinate axis.

$$\psi(M) - \psi(M_o) \simeq -\left(\frac{\partial \psi}{\partial s}\right)_o \cdot s = \sum_{i=1}^{3} \left(\frac{\partial \psi}{\partial x_i}\right)_o \cdot s \cdot \cos(s, x_i) \qquad (2.17)$$

where $\left(\frac{\partial \psi}{\partial s}\right)_o$ is a value of $\frac{\partial \psi}{\partial s}$ at M_o.

Taking into account the expressions (2.2),(2.II), (2.I2) and (2.I7) the equations(2.I3),(2.I4) for $\rho \overline{v_i' v_k'}$ and $\rho \overline{v_i' T'}$ is written in the form

$$-\rho \overline{v_i' v_k'} = -\rho \mathcal{P}_{ik} + \sum_{j=1}^{3} \rho \varepsilon_M^{ij} \frac{\partial \bar{v}_k}{\partial x_j} + \sum_{j=1}^{3} \rho v_M^{kj} \frac{\partial \bar{v}_i}{\partial x_j} \qquad (2.I8)$$

$$\rho \overline{v_i' T'} = \sum_{j=1}^{3} \rho \varepsilon_H^{ij} \frac{\partial \bar{T}}{\partial x}; \qquad (2.I9)$$

where

$$\mathcal{P}_{ik}(M_0) = \mu^2 \int_D L^2 \left|\frac{\partial V}{\partial n}\right|^2 f_0^2 f(M \to M_0) \cos(s, x_i) \cos(s, x_k)\, d\tau, \qquad (2.20)$$

$$\varepsilon_M^{ej}(M_0) = \mu \int_D L \left|\frac{\partial V}{\partial n}\right| f_0 f_1\, s f(M \to M_0) \cos(s, x_e) \cos(s, x_j)\, d\tau, \qquad (2.2I)$$

$$\varepsilon_H^{ej}(M_0) = \mu \int_D L \left|\frac{\partial V}{\partial n}\right| f_0 f_2\, s.f(M \to M_0) \cos(s, x_e) \cos(s, x_j)\, d\tau. \qquad (2.22)$$

The summands in (2.I8) have the same structure as Prandtl's stress in the case of plane flows. And the terms of type $\mathcal{P}_{ik}$ are a turbulent analog of the statical pressure in the fluid. In the case of calculation of practically important problems the system of the equations (I.I-I.3) and (2.I8-2.I9) can in every specific case be considerably simplified if we take into account known hydrodynamical features of the fluid flow considered and the needed accuracy for the solution.

In particular, far from the entrance section the equations of motion (I.I) and of heat inflow for turbulent fluid flows in rectilinear channels can be written after using (2.I8-2.I9) and omitting unessential terms.

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\partial}{\partial x}\left\{(\nu + \varepsilon_M^{11})\frac{\partial w}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{(\nu + \varepsilon_M^{22})\frac{\partial w}{\partial y}\right\}, \qquad (2.23)$$

$$w\frac{\partial T}{\partial z} = \frac{\partial}{\partial x}\left\{(k + \varepsilon_H^{11})\frac{\partial T}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{(k + \varepsilon_H^{22})\frac{\partial T}{\partial y}\right\} + \frac{\partial}{\partial z}\left\{(k + \varepsilon_H^{33})\frac{\partial T}{\partial z}\right\}, \qquad (2.24)$$

where

$$\varepsilon_M^{jj}(M_0) = \mu \int_D L \left|\frac{\partial w}{\partial n}\right| f_0 f_1\, s f(M \to M_0) \cos^2(s, x_j)\, d\tau, \qquad (2.25)$$

$$\varepsilon_H^{jj}(M_0) = \mu \int_D L \left|\frac{\partial w}{\partial n}\right| f_0 f_2\, s f(M \to M_0) \cos^2(s, x_j)\, d\tau, \qquad (2.26)$$

and

$$\left|\frac{\partial w}{\partial n}\right| = \sqrt{\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2}$$

32?

§3. Values of the coefficients μ, α, A_1 and A_2

Determine the empirical coefficients α, μ, A_1 and A_2 included in the discussed model of turbulent transfer by means of some experimental measurements of the correlation moments in turbulent flow and by using other empirical data.

First of all to ensure the agreement between the empirical data and the results of calculations according to (2.18) of the root-mean-square turbulent velocity and the tangential stresses in turbulent fluid flows we must assume

$$\mu = 1,7 \div 1,8 \quad , \qquad \mu\alpha = 0,75. \tag{3.I}$$

The mechanism of the interaction between a lump and the surroundings, described by (2.5) is assumed to be double. Firstly, due to the molecular diffusion the moving lump exchanges the quantities v_i and T with the surroundings. Secondly, the moving lump which is not a solid but some "seething fluid lump" exchanges with the surrounding fluid by "macro-particles".

Therefore, represent the coefficients A_1 and A_2 involved in (2.5) as a sum of two terms corresponding to the action of a molecular mechanism of quantity transfer and to the transfer by "macro-particles":

$$A_1 = (b_1 + b_2)\frac{\nu}{R}, \qquad A_2 = b_3\frac{\kappa}{R} + b_4\frac{\nu}{R}, \tag{3.2}$$

where b_1, b_2, b_3, b_4 are dimensionaless quantities and b_1, b_3 are nearly unities.

Following [4] one can adopt

$$b_4 = b_2, \qquad b_3 = b_1\left(\frac{\nu}{\kappa}\right)^{0,33} \tag{3.3}$$

Taking into account (3.2-3.3) and (2.2) one may write the arguments of functions f_0, f_1 and f_2 in the form

$$P_1 S = \frac{12\theta^2(b_1+b_2)}{\mu\alpha\cdot\gamma_*}\cdot\frac{S}{\alpha L}, \quad P_2 S = \frac{12\theta^2(b_1\mathcal{D}^{0.67}+b_2)}{\mu\alpha,\gamma_*}\frac{S}{\alpha L}, \tag{3.4}$$

where

$$\gamma_* = \frac{L^2}{\nu}\left|\frac{\partial v}{\partial n}\right|, \quad \mathcal{D} = \frac{\kappa}{\nu}, \quad \theta = \frac{\alpha}{\beta}$$

For the sake of simplicity we shall assume and

to be constant. The coefficient θ which is a factor for b_1 and for b_2 may be introduced into b_1 and b_2 so that we shall assume it to be equal unity.

Following [4] take

$$b_1 = 1, \quad b_2 = 4. \tag{3.5}$$

After some trial calculations of velocity and temperature fields for the fluid flows in a round pipe the values of the coefficients μ, α, b_1 and b_2 finally have been taken

$$\mu = 1{,}8; \quad \alpha = 0{,}42; \quad b_1 = 0{,}9; \quad b_2 = 3{,}8. \tag{3.6}$$

The critical number ω has been taken to be equal to 25.

§4. Results of calculations of the velocity and temperature fields for turbulent fluid flows in rectilinear channels

The developed model of turbulent transfer is used for calculating the velocity and temperature fields in turbulent flows in a circular tube, in ring and flat clearences, in channels of rectangular section and in cells of rod lattice.

In these problems in practical solving the equations of type (2.23) and (2.24) it is used some simplifications of the expressions (2.25) and (2.26) for the coefficients ε_M^{jj} and ε_H^{jj}. So the integral expressions for ε_M^{jj} and ε_H^{jj} over a lateral region $\mathcal{D}$ around some considered point are simplified to integrals over the segment parallel to the axis x_j.

$$\varepsilon_M^{jj}(M_0) = cL_0 \int_{-\infty}^{\infty} L \left|\frac{\partial w}{\partial n}\right| f_0(q\varepsilon_j) f_1(q\varepsilon_j) G(\varepsilon_j)\, d\zeta_j, \tag{4.1}$$

$$\varepsilon_H^{jj}(M_0) = cL_0 \int_{-\infty}^{\infty} L \left(\frac{\partial w}{\partial n}\right) f_0(q\zeta_j) f_1(\lambda q\zeta_j) G(\zeta_j)\, d\zeta_j, \tag{4.2}$$

where $\zeta_j = \frac{x_j - [illegible]}{\alpha L_0}$, $G(\zeta) = \frac{1}{2}|\zeta| e^{-1/2\zeta^2}$, $q = \alpha L_0 P_1$.

$$[illegible] \quad \frac{u\varkappa}{3}, \quad \lambda = 0{,}8 + 0{,}2\mathcal{D}^{0{,}67} \tag{4.3}$$

The function $L(M)$ in sections in infinite rectilinear channels was calculated according to the formula (2.16) which for such channels is reduced to

$$\frac{1}{L(M)} = \frac{1}{2}\int_0^{2\pi} \frac{1}{\ell}\, d f \tag{4.4}$$

379

Approved For Release 2009/08/17 : CIA-RDP88-00904R000100100030-7

where $\ell(f)$ is the distance between the point M and perimeter of the section of the channel in the direction f .

For the 2ℓ -wide channel (or 2ℓ wide annulus) between two parallel plates the formula (4.4) gives (see [4])

$$\frac{L}{\ell} = \left(1 - \frac{z}{2}\right), \quad z = \frac{y}{\ell}, \qquad (4.5)$$

y is a distance between M and one of the plates.
According to (4.4) it is also easy to obtain a formula for the scale $L(M)$ in a section of a tube of radius a :

$$\frac{L}{a} = \frac{1-\zeta^2}{2E(\zeta, \pi/2)},$$

where

$$\zeta = \frac{r}{a}, \quad E(\zeta, f) = \int_0^{\ell} \sqrt{1 - \zeta^2 \sin^2 \alpha}\, d\alpha \qquad (4.6)$$

A curve for the function $L(\zeta)$ for a round tube is given in Fig.2 . It is seen from Fig.2 that the mixing length ℓ calculated by Nikuradse [5] according to the measured velocity profiles in a tube is practically identical (with the accuracy to the constant factor) to the scale L : $\ell = 0{,}44L$.

The quantity $\tilde{a}$ is defined for an arbitrary channel by $\frac{1}{\tilde{a}} = \frac{1}{2\pi} \int_0^{\pi} \frac{1}{\ell_c} df = \frac{1}{\pi L_c}$. (4.7)
where $\ell_c(f)$ is a distance between the centre"channel" and the wall in the direction f is called as the effective channel radius and in the following will be used as a characteristic cross size of a channel when dimensionless hydrodynamical characteristics are built up .

In particular, for a round tube of the radius a one has according to (4.7): $\tilde{a} = a$; for a flat 2ℓ -wide annulus one obtains $\tilde{a} = \frac{\pi}{2}\ell$; for a channel of rectangular section with the sides $2a$ and $2\ell\,(\ell < a)$ one has

$$\tilde{a} = \frac{\pi}{2} \frac{a\ell}{\sqrt{a^2+\ell^2}} \qquad (4.8)$$

Figures 3 and 7 show the results of calculating a mean-over-section dimensionless velocity $v = \frac{u}{v_*}$ and resistance coefficient $f = \frac{8}{\bar{v}^2}$ for a round tube when the dimensionless dynamical parameter $\phi = \frac{a v_*}{\nu}$ is varied in a wide range. Here a is the tube radius and $v_*^2 = \frac{a}{2} \frac{1}{\rho} \left|\frac{\partial P}{\partial z}\right|$.

For $\phi < 32$ ($Re < 500$) the solution for $\bar{v}$ turns into

329

the solution for the laminar regime $\bar{U}=0,25\phi$.

Computed velocity profiles U over the tube radius are shown in Fig.4. It is drawn the conventional curves $U=f\left(\frac{yV_*}{\nu}\right)$ for different Reynolds number Re, where y is the distance from the wall. The picture of branching of curves $U=f\left(\frac{yV_*}{\nu}\right)$ for different Reynolds number R represented in Fig.4 looks like the one corresponding to the experimental data (see, for example, 9).

Computed profiles of turbulence viscosity ε_μ over tube section at different values of Re are represented in Fig.5.

As it is seen from figures 3-5 the results of velocity field computations for tubes are in a good agreement with the experimental data.

Note that in computing the velocity field (as well as the temperature fields in fluid with $Pr \leq 1$) the condition (2.3) is found to be important, however, in computing the temperature fields in fluid flows with $Pr > 1$ the condition (2.3) involved decreases the computed number Nu and gives rise to the more strongly pronounced transition of the solution for Nu to a solution for a laminar flow (for Re 400).

Figure 6 shows the results of computations of the number Nu for flows of different fluids in a round tube when the condition at the tube wall is q =const where q is a heat flux density. In the number Nu intervals from 3.10^3 to 3.10^6 the represented results of computing the number Nu in a round tube may be described by an interpolation formula

$$Nu = A + 3{,}90\,(Re\cdot 10^{-3})^{m}\,Pr^{n}, \tag{4.9}$$

where $A = 2{,}5 + 1{,}3\,\lg(1 + Pr^{-1})$,

$m = 0{,}918 - 0{,}051\,\lg(1 + 10\,Pr^{-1})$

$n = 0{,}65 - 0{,}107\,\lg(1 + 10\,Pr)$

The computation results represented in Fig.6 are in rather good agreement with the experimental data both for fluids with $Pr < 1$([10],[11] etc.) and for the fluids with large number Pr([11], [12] etc.).

Fig.7 shows the results of computing the resistance coefficients $\xi = \frac{8}{\bar{U}^2}$ for annular gaps formed by two coaxial cylinders with the radii a_1 and a_2 ($a_1 < a_2$) at different values of the dynamical parameter $\phi = \frac{\tilde{a}\, v_*}{\nu}$ (or at different $Re = \frac{2\tilde{a}\,\bar{u}}{\nu}$)

where $v_*^2 = \frac{\tilde{a}}{2} \cdot \frac{1}{\rho} \left| \frac{\partial P}{\partial z} \right|$, $U = \frac{u}{v_*}$,

and

$$\tilde{a} = \left[\frac{\pi}{4} + \left(1 - \frac{\pi}{4}\right)\left(1 - \frac{a_1}{a_2}\right)^2 \right] (a_2 - a_1) \quad ^{*)} \qquad (4.10)$$

As it is seen from Fig. 7 the curves for $\xi = f(Re)$ in annular gaps at different parameters $\theta = \frac{a_1}{a_2}$ are between the corresponding curves $\xi = f(Re)$ for the round tube and for the flat clearance.

When ϕ is decreasing the solution $\bar{U}$ for a flat annulus in chosen variables is approached to the formula $\bar{U} = 0{,}27\,\phi$

In Fig.9 an example of computing the velocity field $U(x,y)$ in a channel of rectangular section with the parameter $\gamma = \frac{a}{b} = 2$ for arbitrary channels $U = \frac{u}{v_*}$, $v_*^2 = \frac{a}{2}\,\frac{1}{\rho}\left|\frac{\partial P}{\partial z}\right|$, $\phi = \frac{\tilde{a}\, v_*}{\nu}$, $Re = \frac{2\tilde{a}\,\bar{a}}{\nu}$, $\xi = \frac{8}{\bar{U}^2}$.

The results of computing the resistance coefficient ξ in turbulent fluid flows in the channels of the rectangular section are represented in Fig.8 at different values of the relation of the section size γ one to another. For a given value of Reynolds number the resistance coefficient ξ decreases with the parameter increasing.

The results of measurements of the resistance coefficients ξ in the channels of rectangular section are represented in Fig8 I5 . As follows from Fig.8 the results of the calculations of the resistance coefficients in channels of rectangular section are in rather good agreement with the results of the measurements.

In view of the fact that the temperature field in a fluid flow in a channel of the rectangular section is determined by a great number of parameters such as: the heat

---

* The formula (4.I0) is an interpolating one

329

source distribution , the design and the heat-conductivity of channel walls, the number Pr for the fluid and the numbers Re and j, it is not necessary to construct any generalized characteristics of the temperature field as functions of the numbers Re, Pr etc*

It is worth to calculate the temperature fields using (2.24) only for a given design of heat-exchanger cell.

Give now some results of velocity and temperature field calculations for turbulent fluid flows in rod lattices. In Fig.IO an example of the velocity field calculation for the triangular array with the relative spacing h =I.2 is given for the case of a longitudinal flowing around of this array.

The results of the resistance coefficient calculations for the triangular array as functions of Re and at the relative spacing h are represented in Fig.II.

When the characteristic cross size of the channel $\tilde{a}$ has been taken here, the computed curves $\xi = f(Re, h)$ for different values of relative spacing h are in a logical sequence. At equal values of the number Re the resistance coefficients ξ increase with deviating the channel section shape from the circle one.

When however one uses the hydraulic radius R' as a defining cross-size of channel the obtained generalized dependence $\xi' = f(Re', h)$ has no such logical regularuty.

Fig.II shows also the resistance coefficient calculation results for the cell of rectangular lattice of rods with the relative spacing h =I.0. Here the results of, resistance coefficient measurements for the cells of different

---

*) The values of the coefficient ξ for rectangular channels computed in [I5] using Daysler-Taylor graph-analitical method are close to the values ξ obtained in this paper.

329

lattices taken from [I4] and [I3] are shown by the dotted curves. As it is seen from Fig.II, the resistance coefficients computed in this paper for the lattices with the relative lattice spacing h =I.0-I.2 are in satisfactory agreement with the experimental results.

Fig.I2 shows an example of computing the dimensionless temperature $\Theta = \frac{\lambda_m (T - T_c)}{2 q_0 R}$ on surface of the fuel rod in the cell of triangular array with h =I.0 when the fluid with Pr =0.025 flows through this cell . The number Re is equal to 5.IO^4. Here the rod section at the circle with radius r_0 =0.773.R where R is the rod radius. The problem conditions here correspond to the physical ones for the experiments [I6] . In Fig.I2 the dotted line represents the results of measurements of the function Θ from [I6] .

It is seen from Fig.I2 that the results of computing the temperature Θ for these specific conditions are in a rather good agreement with the experimental ones.

Thus, the results of calculation of the velocity and temperature fields in the fluid flows in different channels established in this paper allow to think that the later model of turbulent transfer represented may be used for computing the hydraulic resistances and thermal regime in the cells of heat-exchanger of arbitrary design.

329

REFERENCES

I. O. Reynolds, Phil.Trans.of the Royal Society, I895.

2. L.G. Loytsyanski, Fluid and Gas Mechanics, Moscow I959.

3. L.Prandtl, V.D.I., 77, No 5, I933.

4. N.I.Buleev, Coll. "Heat Transfer" edited by AN USSR, I962

5. I. Nikuradze, Forschungsheft, 356, I932.

6. M.E. Minskii, AN USSR, 28, No 8, I940.

7. K.D. Voskresenskii, E.S.Turilina, Coll."Heat Transfer and Heat Modelling" edited by AN USSR, I959.

8. R.O.Doissler, NAGA, R.M. E52F05, I95I.

9. R.R.Rothfuns and C.C.Monrad, Industr.and Eng.Chemistry, v. 47, No 6, I955.

I0. V.I.Subbotin, P.A.Ushakov, B.N.Gabrionovitch et al., Eng.Phys.Journal No 4, I963, USSR.

II. M.A.Mikheev, "Heat Transfer Fundamentals", Moscow, I956

I2. W. Buhne, Die Warme, v.6I, No 9, I938.

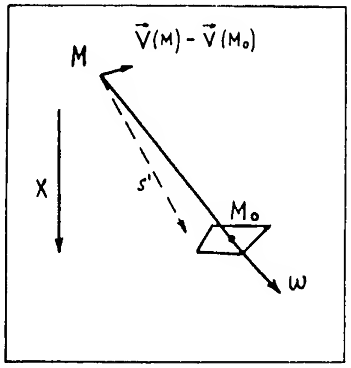
I3. B.M.Teurneau, R.E.Grimble and J.E.Zerbe, Trans. ASME, v. 79, No 8, I957.

I4. V.I.Subbotin, P.A.Ushakov, B.N.Gabrionovitch et al., "Atomnaya Energiya" v.I3,No 2, I962, USSR.

I5. I.P.Hartuett, I.C.Y. Koh, S.T. Mccomas, Trans.ASME, Ser.C, No I, I962.

I6. V.I.Subbotin, P.A.Ushakov, B.N.Gabrionovitch, A.V.Zhukov, "Atomnaya Energiya" v.9, No 6, I960, USSR.

329



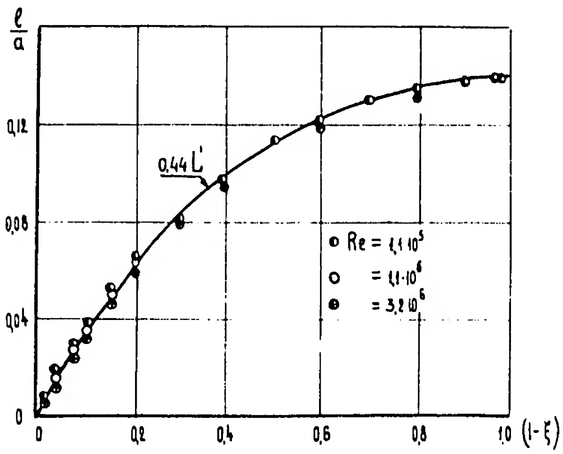


Fig.2 Comparison of the function L and ℓ for the circular pipe

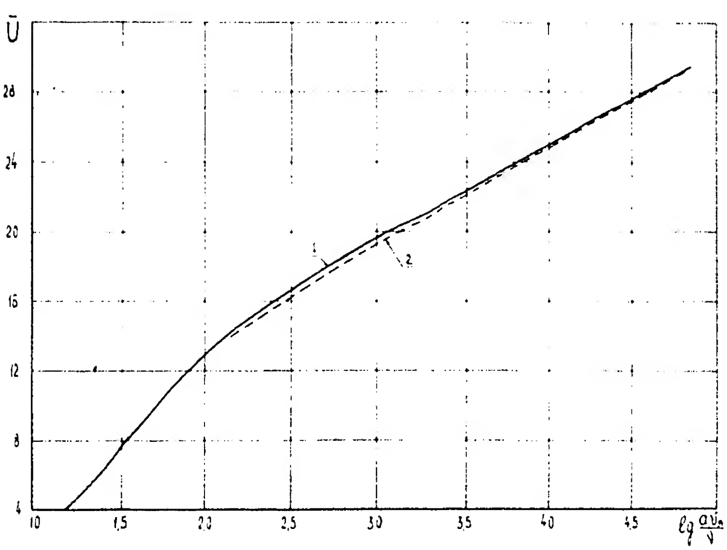


Fig.3 The mean velocity $\bar{U}$ vs the dynamical parameter Φ for circular pipe
I-compound curve,
2-experimental curve

329

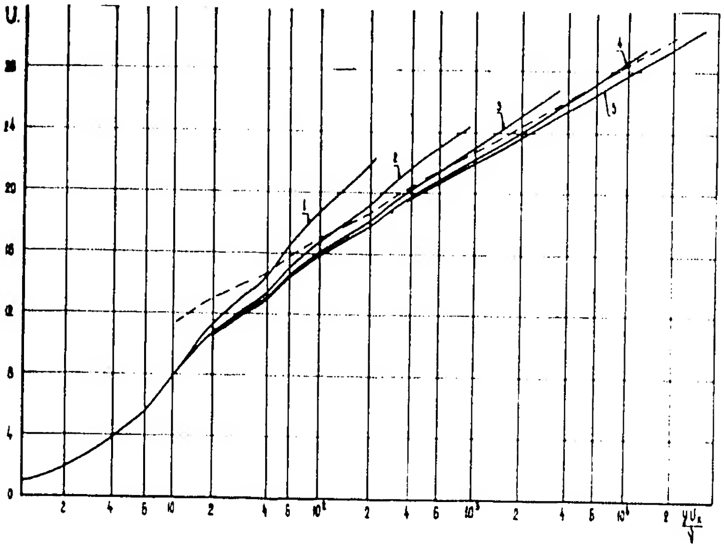


Fig.4 Computed velocity profiles $v(y)$ in a circular pipe at different values of number Re .
I-5 correspond to Re= $=6.9\cdot10^3$, $3.4\cdot10^4$, $I.6\cdot10^5$, $3.2\cdot10^6$.
Dotted curve corresponds to Prandtl solution:
$v = 5{,}5 + 5{,}75 \lg \frac{y v_*}{\nu}$.

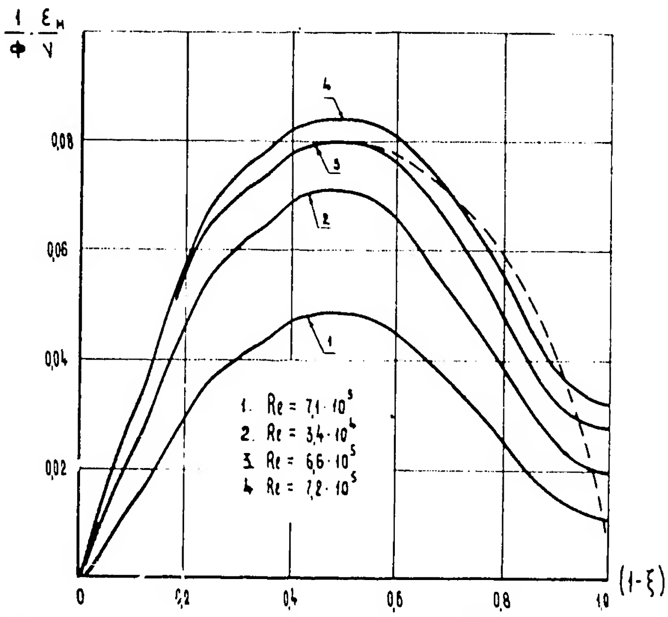


Fig.5 Computed turbulent viscosities ε_H in pipe fluid flow I-4 correspond to Re= $6.9\cdot10^3$, $3.4\cdot10^4$, $I.6\cdot10^5$ $7.3\cdot10^5$.
Dotted curve correspond to the experimental data[5] for Re=10^5 - 10^6.

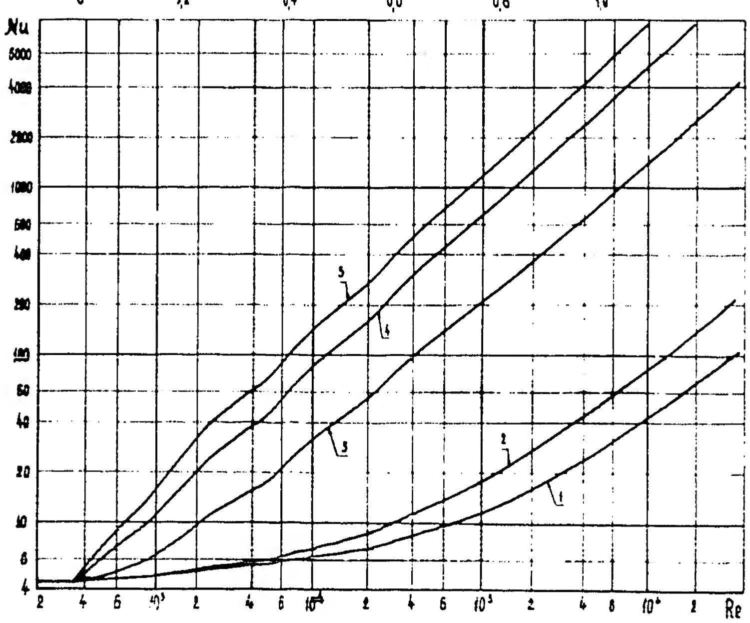


Fig.6 Computed numbers of Nu for fluid flows in circular pipe.
I-Pr=0.0IO,2-Pr=0.025, 3-Pr=I.0, 4-Pr=IO, 5-Pr=IOO.

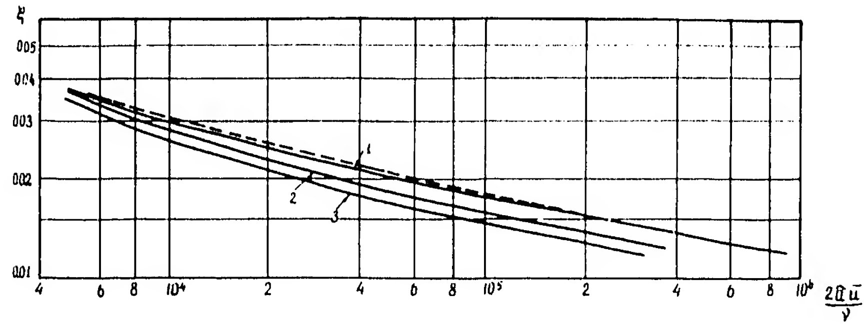


Fig.7 Computed resistance coefficients ξ for ring and flat clearances. I-circular pipe ($\Theta = 0$) . 2-ring clearance with Θ =0.5; 3-clearance with Θ =0.8-I.0; – – – – experimental data for circular pipe.

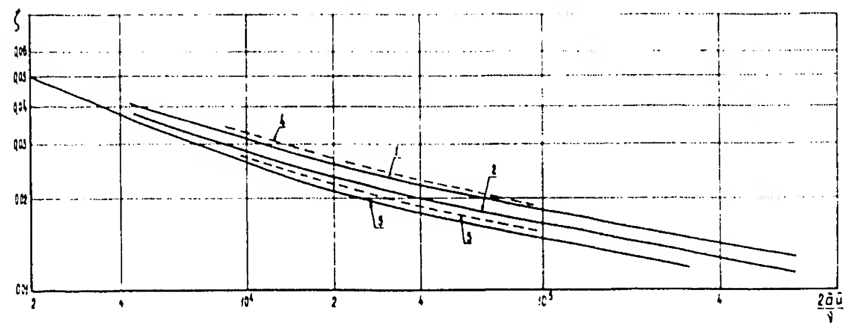


Fig.8 Computed resistance coefficients ξ for the channels of rectangular section. I-2 correspond to γ =I and γ =5. flat clearance; 4,5- are the measurement results [I5] for γ =I and γ=5÷IO.

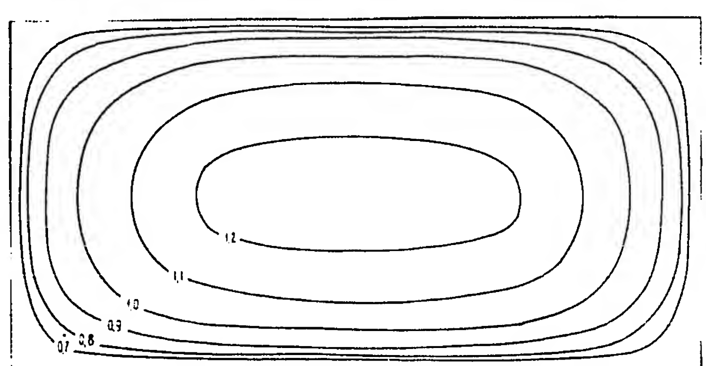


Fig. 9 Computed velocity field $v(x,y)$ in channels of rectangular section. γ =2, Re=3.4xI0^4.

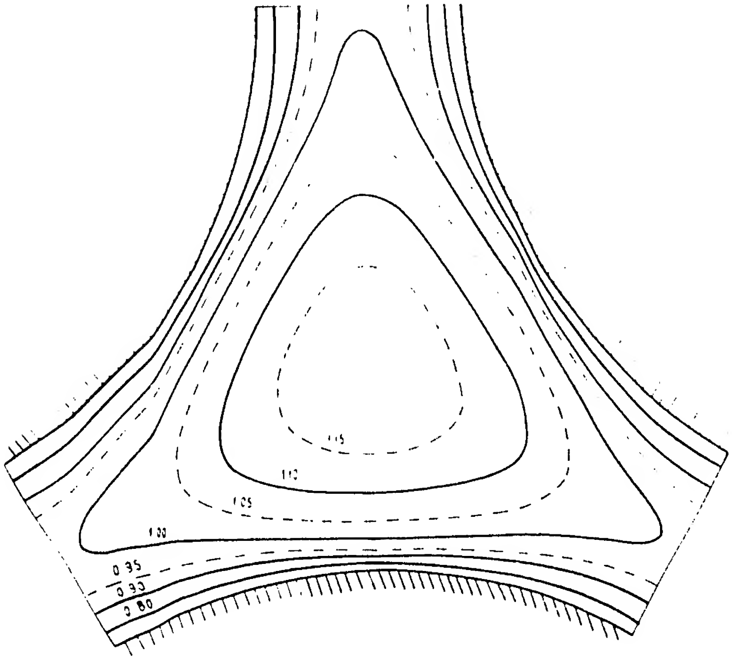


Fig.IO Computed velocity field $v(r,\varphi)$ for the cell of triangular array of rods with the relative lattice spacing h =I.2;
Re= 2.48.I0^{4}.

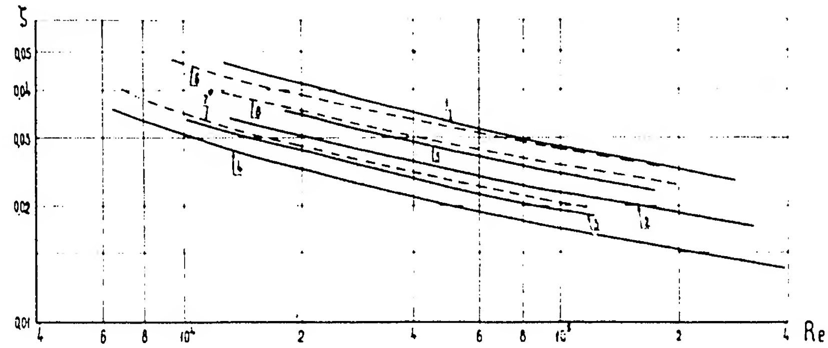


Fig.II Computed resistance coefficients ξ for the cells of triangular and quadratic array of rods for different lattice spacing h . I,2,3-triangular array h=I.0; I.I; I.2; 4-circular pipe; 5-the quadratic lattice cell with h=I.O. Experimental curves. 6,7-triangular array with h=I.0 I4 and h=I.2 [I3] . 8-quadratic array with h=I.0 [I4] .

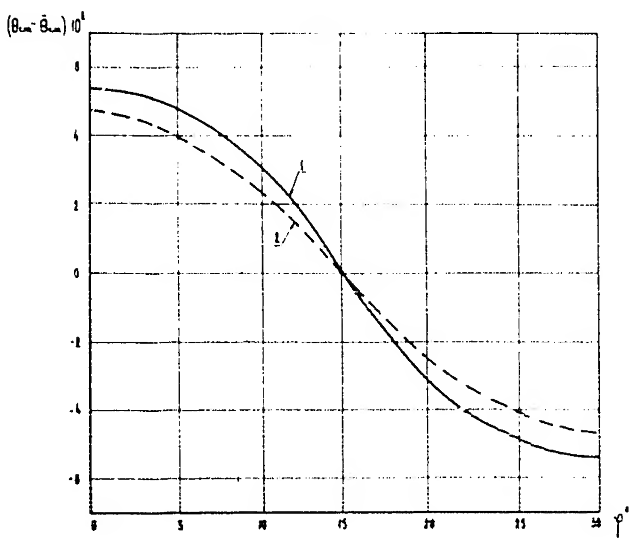


Fig.I2 An example of computing the dimensionless temperature $\Theta = \frac{\lambda_m (T-T_c)}{r q_0 R}$ at the surface of fuel rod in the cell of triangle lattice with the spacing h=I.0; Pr=0.025; Re=5.I0^{4}.

The additional explanations are given in the text;
I-the results of the calculation
2-the results of the measurements [I6].

329